CIS 530: Logistic Regression Wrap-up

SPEECH AND LANGUAGE PROCESSING (3RD EDITION DRAFT)

CHAPTER 5 “LOGISTIC REGRESSION”
Reminders

HW 2 is due tonight before 11:59pm.

Leaderboards are live until then!

Read Textbook
Chapters 3 and 5
Review: Logistic Regression Classifier

For binary text classification, consider an input document $x$, represented by a vector of features $[x_1, x_2, ..., x_n]$. The classifier output $y$ can be 1 or 0.

We want to estimate $P(y = 1 | x)$.

Logistic regression solves this task by learning a vector of weights and a bias term.

$$z = \sum_i w_i x_i + b$$

We can also write this as a dot product:

$$z = w \cdot x + b$$
## Review: Dot product

### Table of Variables

<table>
<thead>
<tr>
<th>Var</th>
<th>Definition</th>
<th>Value</th>
<th>Weight</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>Count of positive lexicon words</td>
<td>3</td>
<td>2.5</td>
<td>7.5</td>
</tr>
<tr>
<td>$x_2$</td>
<td>Count of negative lexicon words</td>
<td>2</td>
<td>-5.0</td>
<td>-10</td>
</tr>
<tr>
<td>$x_3$</td>
<td>Does no appear? (binary feature)</td>
<td>1</td>
<td>-1.2</td>
<td>-1.2</td>
</tr>
<tr>
<td>$x_4$</td>
<td>Num 1\textsuperscript{st} and 2nd person pronouns</td>
<td>3</td>
<td>0.5</td>
<td>1.5</td>
</tr>
<tr>
<td>$x_5$</td>
<td>Does ! appear? (binary feature)</td>
<td>0</td>
<td>2.0</td>
<td>0</td>
</tr>
<tr>
<td>$x_6$</td>
<td>Log of the word count for the doc</td>
<td>4.15</td>
<td>0.7</td>
<td>2.905</td>
</tr>
<tr>
<td>b</td>
<td>bias</td>
<td>1</td>
<td>0.1</td>
<td>.1</td>
</tr>
</tbody>
</table>

\[ z = \sum_i w_i x_i + b \]

\[ z = 0.805 \]
### Review: Sigmoid

<table>
<thead>
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<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

$$\sigma(0.805) = 0.69$$
Review: Learning

How do we get the weights of the model? We learn the parameters (weights + bias) via learning. This requires 2 components:

1. An objective function or **loss function** that tells us *distance* between the system output and the gold output. We use **cross-entropy loss**.

2. An algorithm for optimizing the objective function. We will use stochastic gradient descent to **minimize** the **loss function**. (We’ll cover SGD later when we get to neural networks).
Review: Cross-entropy loss

Why does minimizing this negative log probability do what we want? We want the loss to be smaller if the model’s estimate is close to correct, and we want the loss to be bigger if it is confused.

It's hokey. There are virtually no surprises, and the writing is second-rate. So why was it so enjoyable? For one thing, the cast is great. Another nice touch is the music. I was overcome with the urge to get off the couch and start dancing. It sucked me in, and it'll do the same to you.

\[
P(\text{sentiment}=1|\text{It's hokey...}) = 0.69. \quad \text{Let's say } y=1.
\]

\[
L_{CE}(\hat{y}, y) = -[y \log \sigma(w \cdot x + b) + (1 - y) \log(1 - \sigma(w \cdot x + b))]
\]

\[
= -[\log \sigma(w \cdot x + b)]
\]

\[
= - \log (0.69) = 0.37
\]
Review: Cross-entropy loss

Why does minimizing this negative log probability do what we want? We want the loss to be smaller if the model’s estimate is close to correct, and we want the loss to be bigger if it is confused.

It's hokey. There are virtually no surprises, and the writing is second-rate. So why was it so enjoyable? For one thing, the cast is great. Another nice touch is the music. I was overcome with the urge to get off the couch and start dancing. It sucked me in, and it'll do the same to you.

\[
P(\text{sentiment}=1|\text{It's hokey...}) = 0.69. \quad \text{Let's pretend } y=0.
\]

\[
L_{CE}(\hat{y}, y) = -[y \log \sigma(w \cdot x + b) + (1 - y) \log(1 - \sigma(w \cdot x + b))]
\]

\[
= -[\log(1 - \sigma(w \cdot x + b))]
\]

\[
= - \log(0.31) = 1.17
\]
Loss on all training examples

\[
\log p(\text{training labels}) = \log \prod_{i=1}^{m} p(y^{(i)} | x^{(i)}) \\
= \sum_{i=1}^{m} \log p(y^{(i)} | x^{(i)}) \\
= -\sum_{i=1}^{m} \text{L}_{\text{CE}}(\hat{y}^{(i)} | y^{(i)})
\]
Finding good parameters

We use gradient descent to find good settings for our weights and bias by minimizing the loss function.

$$\hat{\theta} = \arg \min_{\theta} \frac{1}{m} \sum_{i=1}^{m} L_{CE}(y^{(i)}, x^{(i)}; \theta)$$

Gradient descent is a method that finds a minimum of a function by figuring out in which direction (in the space of the parameters $\theta$) the function’s slope is rising the most steeply, and moving in the opposite direction.
Gradient Descent
iOS Autocomplete Song | Song A Day #2110

https://www.youtube.com/watch?v=M8MJFrdfGe0
Probabilistic Language Models

- Autocomplete for texting
- Machine Translation
- Spelling Correction
- Speech Recognition
- Other NLG tasks: summarization, question-answering, dialog systems
Goal: compute the probability of a sentence or sequence of words

Related task: probability of an upcoming word

A model that computes either of these is a language model

Better: the grammar

But LM is standard in NLP
Goal: compute the probability of a sentence or sequence of words
\[ P(W) = P(w_1, w_2, w_3, w_4, w_5 \ldots w_n) \]

Related task: probability of an upcoming word
\[ P(w_5 | w_1, w_2, w_3, w_4) \]

A model that computes either of these \( P(W) \) or \( P(w_n | w_1, w_2 \ldots w_{n-1}) \) is called a **language model**.

Better: **the grammar**

But **language model** or **LM** is standard
How to compute $P(W)$

How to compute this joint probability:

- $P(\text{the, underdog, Philadelphia, Eagles, won})$

Intuition: let’s rely on the Chain Rule of Probability
The Chain Rule
The Chain Rule

Recall the definition of conditional probabilities

\[ p(B|A) = \frac{P(A,B)}{P(A)} \]

Rewriting: \[ P(A,B) = P(A)P(B|A) \]

More variables:

\[ P(A,B,C,D) = P(A)P(B|A)P(C|A,B)P(D|A,B,C) \]

The Chain Rule in General

\[ P(x_1, x_2, x_3, ..., x_n) = P(x_1)P(x_2|x_1)P(x_3|x_1, x_2) ... P(x_n|x_1, ..., x_{n-1}) \]
The Chain Rule applied to compute joint probability of words in sentence
The Chain Rule applied to compute joint probability of words in sentence

\[ P(w_1w_2 \cdots w_n) = \prod_i P(w_i|w_1w_2 \cdots w_{i-1}) \]

P(“the underdog Philadelphia Eagles won”) =
\[ P(\text{the}) \times P(\text{underdog}|\text{the}) \times P(\text{Philadelphia}|\text{the underdog}) \]
\[ \times P(\text{Eagles}|\text{the underdog Philadelphia}) \]
\[ \times P(\text{won}|\text{the underdog Philadelphia Eagles}) \]
How to estimate these probabilities

Could we just count and divide?
How to estimate these probabilities

Could we just count and divide? Maximum likelihood estimation (MLE)

\[ P(\text{won}|\text{the underdog team}) = \frac{\text{Count(\text{the underdog team won})}}{\text{Count(\text{the underdog team})}} \]

Why doesn’t this work?
Simplifying Assumption = Markov Assumption
Simplifying Assumption = Markov Assumption

\[ P(\text{won}|\text{the underdog team}) \approx P(\text{won}|\text{team}) \]

Only depends on the previous \( k \) words, not the whole context

\[ \approx P(\text{won}|\text{underdog team}) \]

\[ \approx P(w_i|w_{i-2} w_{i-1}) \]

\[ P(w_1 w_2 w_3 w_4 ... w_n) \approx \prod_i^n P(w_i|w_{i-k} ... w_{i-1}) \]

\( K \) is the number of context words that we take into account
How much history should we use?

<table>
<thead>
<tr>
<th>Model</th>
<th>History</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unigram</td>
<td>no history</td>
<td>[ p(w_i) = \frac{\text{count}(w_i)}{\text{all words}} ]</td>
</tr>
<tr>
<td>Bigram</td>
<td>1 word as history</td>
<td>[ p(w_i</td>
</tr>
<tr>
<td>Trigram</td>
<td>2 words as history</td>
<td>[ p(w_i</td>
</tr>
<tr>
<td>4-gram</td>
<td>3 words as history</td>
<td>[ p(w_i</td>
</tr>
</tbody>
</table>
# Historical Notes

<table>
<thead>
<tr>
<th>Year</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>1913</td>
<td>Andrei Markov counts 20k letters in <em>Eugene Onegin</em></td>
</tr>
<tr>
<td>1948</td>
<td>Claude Shannon uses n-grams to approximate English</td>
</tr>
<tr>
<td>1956</td>
<td>Noam Chomsky decries finite-state Markov Models</td>
</tr>
<tr>
<td>1980s</td>
<td>Fred Jelinek at IBM TJ Watson uses n-grams for ASR, think about 2 other ideas for models: (1) MT, (2) stock market prediction</td>
</tr>
<tr>
<td>1993</td>
<td>Jelinek at team develops statistical machine translation</td>
</tr>
<tr>
<td></td>
<td>$\argmax_e p(e</td>
</tr>
<tr>
<td></td>
<td>Jelinek left IBM to found CLSP at JHU</td>
</tr>
<tr>
<td></td>
<td>Peter Brown and Robert Mercer move to Renaissance Technology</td>
</tr>
</tbody>
</table>
Simplest case: Unigram model

\[ P(w_1|w_2 \ldots w_n) = \prod_i P(w_i) \]

Some automatically generated sentences from a unigram model

fifth an of futures the an incorporated a a the inflation most dollars quarter in is mass

thrift did eighty said hard 'm july bullish

that or limited the
Bigram model

- Condition on the previous word:

\[ P(w_i|w_1w_2 \ldots w_{i-1}) = P(w_i|w_{i-1}) \]

texaco rose one in this issue is pursuing growth in a boiler house said mr. gurria mexico 's motion control proposal without permission from five hundred fifty five yen outside new car parking lot of the agreement reached
this would be a record november
N-gram models

We can extend to trigrams, 4-grams, 5-grams

In general this is an insufficient model of language
  ◦ because language has long-distance dependencies:

  “The computer(s) which I had just put into the machine room on the fifth floor is (are) crashing.”

But we can often get away with N-gram models
Language Modeling

ESTIMATING N-GRAM PROBABILITIES
Estimating bigram probabilities

The Maximum Likelihood Estimate

\[ P(w_i | w_{i-1}) = \frac{\text{count}(w_{i-1}, w_i)}{\text{count}(w_{i-1})} \]

\[ P(w_i | w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})} \]
An example

\[ P(w_i|w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})} \]

\(<s>\) I am Sam \(<s>\)

\(<s>\) Sam I am \(<s>\)

\(<s>\) I do not like green eggs and ham \(<s>\)

\[ P(I|<s>) = \frac{2}{3} = .67 \]

\[ P(Sam|<s>) = \frac{1}{3} = .33 \]

\[ P(\text{am}|I) = \frac{2}{3} = .67 \]

\[ P(</s>|\text{Sam}) = \frac{1}{2} = 0.5 \]

\[ P(Sam|\text{am}) = \frac{1}{2} = .5 \]

\[ P(\text{do}|I) = \frac{1}{3} = .33 \]
Problems for MLE

Zeros

<table>
<thead>
<tr>
<th>Train</th>
<th>Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>denied the allegations</td>
<td>denied the memo</td>
</tr>
<tr>
<td>denied the reports</td>
<td></td>
</tr>
<tr>
<td>denied the claims</td>
<td></td>
</tr>
<tr>
<td>denied the requests</td>
<td></td>
</tr>
</tbody>
</table>

\[ P(\text{memo}|\text{denied the}) = 0 \]

And we also assign 0 probability to all sentences containing it!
Problems for MLE

Out of vocabulary items (OOV)

<unk> to deal with OOVs

Fixed lexicon $L$ of size $V$

Normalize training data by replacing any word not in $L$ with <unk>

Avoid zeros with smoothing
Practical Issues

We do everything in log space

- Avoid underflow
- (also adding is faster than multiplying)

$$\log(p_1 \cdot p_2 \cdot p_3 \cdot p_4) = \log p_1 + \log p_2 + \log p_3 + \log p_4$$
Language Modeling Toolkits

SRILM

KenLM
- [https://kheafield.com/code/kenlm/](https://kheafield.com/code/kenlm/)
All Our N-gram are Belong to You

Posted by Alex Franz and Thorsten Brants, Google Machine Translation Team

Here at Google Research we have been using word n-gram models for a variety of R&D projects, ...

That's why we decided to share this enormous dataset with everyone. We processed 1,024,908,267,229 words of running text and are publishing the counts for all 1,176,470,663 five-word sequences that appear at least 40 times. There are 13,588,391 unique words, after discarding words that appear less than 200 times.

https://ai.googleblog.com/2006/08/all-our-n-gram-are-belong-to-you.html
Google N-Gram Release

serve as the incoming 92
serve as the incubator 99
serve as the independent 794
serve as the index 223
serve as the indication 72
serve as the indicator 120
serve as the indicators 45
serve as the indispensable 111
serve as the indispensible 40
serve as the individual 234

https://ai.googleblog.com/2006/08/all-our-n-gram-are-belong-to-you.html
Google Book N-grams

https://books.google.com/ngrams
Language Modeling

EVALUATION AND PERPLEXITY
Evaluation: How good is our model?

Does our language model prefer good sentences to bad ones?
- Assign higher probability to “real” or “frequently observed” sentences
  - Than “ungrammatical” or “rarely observed” sentences?

We train parameters of our model on a training set.

We test the model’s performance on data we haven’t seen.
- A test set is an unseen dataset that is different from our training set, totally unused.
- An evaluation metric tells us how well our model does on the test set.
We can’t allow test sentences into the training set

We will assign it an artificially high probability when we set it in the test set

“Training on the test set”

Bad science!

And violates the honor code
Extrinsic evaluation of language models
Extrinsic evaluation
- Time-consuming; can take days or weeks

So
- Sometimes use intrinsic evaluation: perplexity
- Bad approximation
  - unless the test data looks just like the training data
  - So generally only useful in pilot experiments
- But is helpful to think about.
Intuition of Perplexity

The Shannon Game:
- How well can we predict the next word?

I always order pizza with cheese and ____
Intuition of Perplexity

The Shannon Game:
- How well can we predict the next word?

I always order pizza with cheese and ____

- mushrooms 0.1
- pepperoni 0.1
- anchovies 0.01
- ....
- fried rice 0.0001
- ....
- and 1e-100
Intuition of Perplexity

The Shannon Game:

- How well can we predict the next word?

I always order pizza with cheese and ____

The 33rd President of the US was ____

I saw a ____

- Unigrams are terrible at this game. (Why?)

<p>| | |</p>
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<tr>
<td>mushrooms</td>
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<tr>
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<td>0.1</td>
</tr>
<tr>
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<td>....</td>
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</tr>
<tr>
<td>and 1e-100</td>
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Intuition of Perplexity

The Shannon Game:

◦ How well can we predict the next word?

I always order pizza with cheese and ____
The 33rd President of the US was ____
I saw a ____

{ mushrooms 0.1
  pepperoni 0.1
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  ....
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  ....
  and 1e-100 }
Intuition of Perplexity

The Shannon Game:
- How well can we predict the next word?

- I always order pizza with cheese and ____

- The 33\textsuperscript{rd} President of the US was ____

- I saw a ____

- Unigrams are terrible at this game. (Why?)

A better model of a text
- is one which assigns a higher probability to the word that actually occurs

- mushrooms 0.1
- pepperoni 0.1
- anchovies 0.01
- ....
- fried rice 0.0001
- ....
- and 1e-100
Perplexity

The best language model is one that best predicts an unseen test set
  • Gives the highest $\text{P}(\text{sentence})$

Perplexity is the inverse probability of the test set, normalized by the number of words

Minimizing perplexity is the same as maximizing probability
Perplexity

The best language model is one that best predicts an unseen test set

- Gives the highest $P($sentence$)$

Perplexity is the inverse probability of the test set, normalized by the number of words

Perplexity is defined as:

$$PP(W) = P(w_1 w_2 \cdots w_N)^{-\frac{1}{N}}$$

Chain rule:

$$PP(W) = \sqrt[N]{\frac{1}{\prod_{i=1}^{N} P(w_i|w_1, w_2 \cdots w_{i-1})}}$$

For bigrams:

$$PP(W) = \sqrt[N]{\prod_{i=1}^{N} \frac{1}{P(w_i|w_{i-1})}}$$

Minimizing perplexity is the same as maximizing probability
Perplexity as branching factor

Let’s suppose a sentence consisting of random digits

What is the perplexity of this sentence according to a model that assigns $P=1/10$ to each digit?
Perplexity as branching factor

Let’s suppose a sentence consisting of random digits

What is the perplexity of this sentence according to a model that assign $P=1/10$ to each digit?

$$PP(W) = P(w_1w_2 \cdots w_N)^{-\frac{1}{N}}$$

$$= \left(\frac{1}{10}\right)^{N\frac{1}{N}}$$

$$= \frac{1}{10}^{-1}$$

$$= \frac{10}{10}$$

$$= 10$$
Lower perplexity = better model

Minimizing perplexity is the same as maximizing probability

Training 38 million words, test 1.5 million words, WSJ

<table>
<thead>
<tr>
<th>N-gram Order</th>
<th>Unigram</th>
<th>Bigram</th>
<th>Trigram</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perplexity</td>
<td>962</td>
<td>170</td>
<td>109</td>
</tr>
</tbody>
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Language Modeling

GENERALIZATION AND ZEROS
The Shannon Visualization Method

Choose a random bigram

\(<s>, w)\) according to its probability

Now choose a random bigram \((w, x)\) according to its probability

And so on until we choose \(<s>\)

Then string the words together

\(<s>\ I
I \ want
want \ to
\ to \ eat
eat \ Chinese
Chinese \ food
\ food \ <s>\n
I want to eat Chinese food
Imagine all the words of English covering the probability space between 0 and 1, each word covering an interval proportional to its frequency. We choose a random value between 0 and 1 and print the word whose interval includes this chosen value. We continue choosing random numbers and generating words until we randomly generate the sentence-final token `.</s>`. We can use the same technique to generate bigrams by first generating a random bigram that starts with `<s>` (according to its bigram probability), then choosing a random bigram to follow (again, according to its bigram probability), and so on.

To give an intuition for the increasing power of higher-order N-grams, Fig. 4.3 shows random sentences generated from unigram, bigram, trigram, and 4-gram models trained on Shakespeare’s works.

| 1 gram | –To him swallowed confess hear both. Which. Of save on trail for are ay device and rote life have |
| 2 gram | –Hill he late speaks; or! a more to leg less first you enter |
| 3 gram | –Why dost stand forth thy canopy, forsooth; he is this palpable hit the King Henry. Live king. Follow. |
| 4 gram | –What means, sir. I confess she? then all sorts, he is trim, captain. |
|       | –Fly, and will rid me these news of price. Therefore the sadness of parting, as they say, ’tis done. |
|       | –This shall forbid it should be branded, if renown made it empty. |
|       | –It cannot be but so. |
|       | –King Henry. What! I will go seek the traitor Gloucester. Exeunt some of the watch. A great banquet serv’d in; |

The longer the context on which we train the model, the more coherent the sentences. In the unigram sentences, there is no coherent relation between words or any sentence-final punctuation. The bigram sentences have some local word-to-word coherence (especially if we consider that punctuation counts as a word). The trigram and 4-gram sentences are beginning to look a lot like Shakespeare. Indeed, a careful investigation of the 4-gram sentences shows that they look a little too much like Shakespeare. The words `It cannot be but so` are directly from `King John`. This is because, not to put the knock on Shakespeare, his oeuvre is not very large as corpora go, \( N = 884,647, V = 29,066 \), and our N-gram probability matrices are ridiculously sparse. There are \( V^2 = 844,000,000 \) possible bigrams alone, and the number of possible 4-grams is \( V^4 = 7 \times 10^{17} \). Thus, once the generator has chosen the first 4-gram (`It cannot be but`), there are only five possible continuations (`that`, `I`, `he`, `thou`, and `so`); indeed, for many 4-grams, there is only one continuation.

To get an idea of the dependence of a grammar on its training set, let’s look at an N-gram grammar trained on a completely different corpus: the `Wall Street Journal` (WSJ) newspaper. Shakespeare and the `Wall Street Journal` are both English, so we might expect some overlap between our N-grams for the two genres. Fig. 4.4 shows sentences generated by unigram, bigram, and trigram grammars trained on 40 million words from WSJ.

Compare these examples to the pseudo-Shakespeare in Fig. 4.3. While superficially they both seem to model “English-like sentences”, there is obviously no overlap.
Shakespeare as corpus

V=29,066 types, N=884,647 tokens
Shakespeare as corpus

N=884,647 tokens, V=29,066

Shakespeare produced 300,000 bigram types out of $V^2=844$ million possible bigrams.

- So 99.96% of the possible bigrams were never seen (have zero entries in the table)

4-grams worse: What's coming out looks like Shakespeare because it is Shakespeare
The wall street journal is not shakespeare (no offense)

1
gram

Months the my and issue of year foreign new exchange’s september were recession exchange new endorsed a acquire to six executives

2
gram

Last December through the way to preserve the Hudson corporation N. B. E. C. Taylor would seem to complete the major central planners one point five percent of U. S. E. has already old M. X. corporation of living on information such as more frequently fishing to keep her

3
gram

They also point to ninety nine point six billion dollars from two hundred four oh six three percent of the rates of interest stores as Mexico and Brazil on market conditions
Can you guess the author of these random 3-gram sentences?

They also point to ninety nine point six billion dollars from two hundred four oh six three percent of the rates of interest stores as Mexico and gram Brazil on market conditions

This shall forbid it should be branded, if renown made it empty.

“You are uniformly charming!” cried he, with a smile of associating and now and then I bowed and they perceived a chaise and four to wish for.
The perils of overfitting

N-grams only work well for word prediction if the test corpus looks like the training corpus
  ◦ In real life, it often doesn’t
  ◦ We need to train robust models that generalize!
  ◦ One kind of generalization: Zeros!
    ◦ Things that don’t ever occur in the training set
    ◦ But occur in the test set
Zero probability bigrams

Bigrams with zero probability
- mean that we will assign 0 probability to the test set!

And hence we cannot compute perplexity (can’t divide by 0)!
Language Modeling

SMOOTHING: ADD-ONE (LAPLACE) SMOOTHING
The intuition of smoothing (from Dan Klein)

When we have sparse statistics:

\[ P(w \mid \text{denied the}) \]
- 3 allegations
- 2 reports
- 1 claims
- 1 request
- 7 total

Steal probability mass to generalize better

\[ P(w \mid \text{denied the}) \]
- 2.5 allegations
- 1.5 reports
- 0.5 claims
- 0.5 request
- 2 other
- 7 total
Add-one estimation

Also called Laplace smoothing

Pretend we saw each word one more time than we did

Just add one to all the counts!

MLE estimate:

$$P_{MLE}(w_i|w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

Add-1 estimate:

$$P_{Add-1}(w_i|w_{i-1}) = \frac{c(w_{i-1}, w_i) + 1}{c(w_{i-1}) + V}$$
Maximum Likelihood Estimates

The maximum likelihood estimate
  ◦ of some parameter of a model $M$ from a training set $T$
  ◦ maximizes the likelihood of the training set $T$ given the model $M$

Suppose the word “bagel” occurs 400 times in a corpus of a million words

What is the probability that a random word from some other text will be “bagel”? Why is it

MLE estimate is $400/1,000,000 = .0004$

This may be a bad estimate for some other corpus
  ◦ But it is the estimate that makes it most likely that “bagel” will occur 400 times in a million word corpus.
Add-1 estimation is a blunt instrument

So add-1 isn’t used for N-grams:
- We’ll see better methods

But add-1 is used to smooth other NLP models
- For text classification
- In domains where the number of zeros isn’t so huge.
Language Modeling

INTERPOLATION, BACKOFF, AND WEB-SCALE LMS
Sometimes it helps to use **less** context. Condition on less context for contexts you haven’t learned much about.

**Backoff:**
- use trigram if you have good evidence,
  otherwise bigram, otherwise unigram

**Interpolation:**
- mix unigram, bigram, trigram

Interpolation works better
Linear Interpolation

Simple interpolation

\[
\hat{P}(w_n | w_{n-2}w_{n-1}) = \lambda_1 P(w_n | w_{n-2}w_{n-1}) + \lambda_2 P(w_n | w_{n-1}) + \lambda_3 P(w_n)
\]

\[
\sum_i \lambda_i = 1
\]

Lambdas conditional on context:

\[
\hat{P}(w_n | w_{n-2}w_{n-1}) = \lambda_1 \binom{n-1}{n-2} P(w_n | w_{n-2}w_{n-1}) + \lambda_2 \binom{n-1}{n-2} P(w_n | w_{n-1}) + \lambda_3 \binom{n-1}{n-2} P(w_n)
\]
How to set the lambdas?

Use a **held-out** corpus

Choose $\lambda$s to maximize the probability of held-out data:
- Fix the N-gram probabilities (on the training data)
- Then search for $\lambda$s that give largest probability to held-out set:

\[
\log P(w_1 \cdots w_n | M(\lambda_1 \cdots \lambda_k)) = \sum_i \log P_{M(\lambda_1 \cdots \lambda_k)}(w_i | w_{i-1})
\]
Unknown words: Open versus closed vocabulary tasks

If we know all the words in advanced
- Vocabulary V is fixed
- Closed vocabulary task

Often we don’t know this
- **Out Of Vocabulary** = OOV words
- Open vocabulary task

Instead: create an unknown word token `<UNK>`
- Training of `<UNK>` probabilities
  - Create a fixed lexicon L of size V
  - At text normalization phase, any training word not in L changed to `<UNK>`
  - Now we train its probabilities like a normal word
- At decoding time
  - If text input: Use UNK probabilities for any word not in training
Huge web-scale n-grams

How to deal with, e.g., Google N-gram corpus

Pruning
- Only store N-grams with count > threshold.
  - Remove singletons of higher-order n-grams
- Entropy-based pruning

Efficiency
- Efficient data structures like tries
- Bloom filters: approximate language models
- Store words as indexes, not strings
  - Use Huffman coding to fit large numbers of words into two bytes
- Quantize probabilities (4-8 bits instead of 8-byte float)
Smoothing for Web-scale N-grams

“Stupid backoff” (Brants et al. 2007)

No discounting, just use relative frequencies

\[
S(w_i | w_{i-k+1}^{i-1}) = \begin{cases} 
\frac{\text{count}(w_{i-k+1}^i)}{\text{count}(w_{i-k+1}^{i-1})} & \text{if count}(w_{i-k+1}^i) > 0 \\
0.4S(w_i | w_{i-k+2}^{i-1}) & \text{otherwise}
\end{cases}
\]

\[
S(w_i) = \frac{\text{count}(w_i)}{N}
\]
N-gram Smoothing Summary

Add-1 smoothing:
- OK for text categorization, not for language modeling

The most commonly used method:
- Extended Interpolated Kneser-Ney

For very large N-grams like the Web:
- Stupid backoff
Advanced Language Modeling

Discriminative models:
- choose n-gram weights to improve a task, not to fit the training set

Parsing-based models

Caching Models
- Recently used words are more likely to appear

\[
P_{\text{CACHE}}(w|\text{history}) = \lambda P(w_i|w_{i-2}w_{i-1}) + (1 - \lambda) \frac{c(w \in \text{history})}{|\text{history}|}
\]
Vector Space
Semantics

READ CHAPTER 6 IN THE DRAFT 3RD EDITION OF JURAFSKY AND MARTIN
What does ongchoi mean?

Suppose you see these sentences:

Ong choi is delicious sautéed with garlic.
Ong choi is superb over rice
Ong choi leaves with salty sauces

And you've also seen these:

...spinach sautéed with garlic over rice
Chard stems and leaves are delicious
Collard greens and other salty leafy greens

Conclusion:

Ongchoi is a leafy green like spinach, chard, or collard greens
Ong choi: *Ipomoea aquatica*
"Water Spinach"

Yamaguchi, Wikimedia Commons, public domain
If we consider *optometrist* and *eye-doctor* we find that, as our corpus of utterances grows, these two occur in almost the same environments. In contrast, there are many sentence environments in which *optometrist* occurs but *lawyer* does not...

It is a question of the relative frequency of such environments, and of what we will obtain if we ask an informant to substitute any word he wishes for *oculist* (not asking what words have the same meaning).

These and similar tests all.
We'll build a new representation of words that encodes their similarity.

Each word = a vector

Similar words are "nearby in space"
We define a word as a vector

Called an "embedding" because it's embedded into a space

The standard way to represent meaning in NLP

Fine-grained model of meaning for similarity

- NLP tasks like sentiment analysis
  - With words, requires **same** word to be in training and test
  - With embeddings: ok if **similar** words occurred!!!
- Question answering, conversational agents, etc
We'll introduce 2 kinds of embeddings

**Tf-idf**
- A common baseline model
- Sparse vectors
- Words are represented by a simple function of the counts of nearby words

**Word2vec**
- Dense vectors
- Representation is created by training a classifier to distinguish nearby and far-away words