Vector Semantics

Read Chapter 6 of Speech and Language Processing (3\textsuperscript{rd} ed draft)
Why vector models of meaning? Computing the similarity between words

“fast” is similar to “rapid”
“tall” is similar to “height”

Question answering:
Q: “How tall is Mt. Everest?”
Candidate A: “The official height of Mount Everest is 29029 feet”
Word similarity for plagiarism detection

MAINFRAMES
Mainframes are primarily referred to large computers with rapid, advanced processing capabilities that can execute and perform tasks equivalent to many Personal Computers (PCs) machines networked together. It is characterized with high quantity Random Access Memory (RAM), very large secondary storage devices, and high-speed processors to cater for the needs of the computers under its service.

Consisting of advanced components, mainframes have the capability of running multiple large applications required by many and most enterprises and organizations. This is one of its advantages. Mainframes are also suitable to cater for those applications (programs) or files that are of very high

MAINFRAMES
Mainframes usually are referred those computers with fast, advanced processing capabilities that could perform by itself tasks that may require a lot of Personal Computers (PC) Machines. Usually mainframes would have lots of RAMs, very large secondary storage devices, and very fast processors to cater for the needs of those computers under its service.

Due to the advanced components mainframes have, these computers have the capability of running multiple large applications required by most enterprises, which is one of its advantage. Mainframes are also suitable to cater for those applications or files that are of very large demand
Distributional models of meaning = vector-space models of meaning = vector semantics

Intuitions: Zellig Harris (1954):
• “oculist and eye-doctor ... occur in almost the same environments”
• “If A and B have almost identical environments we say that they are synonyms.”

Firth (1957):
• “You shall know a word by the company it keeps!”
Intuition of distributional word similarity

• Nida example:
  A bottle of *tesgüino* is on the table
  Everybody likes *tesgüino*
  *Tsegüino* makes you drunk
  We make *tesgüino* out of corn.

• From context words humans can guess *tesgüino* means
  • an alcoholic beverage like *beer*

• Intuition for algorithm:
  • Two words are similar if they have similar word contexts.
Intuition

- Model the meaning of a word by “embedding” in a vector space.
- The meaning of a word is a vector of numbers
  - Vector models are also called “embeddings”.
- Contrast: word meaning is represented in many computational linguistic applications by a vocabulary index (“word number 545”)
- Old philosophy joke:
  
  Q: What’s the meaning of life?
  A: LIFE’
Term-document matrix
Term-document matrix

- Each cell: count of term $t$ in a document $d$: $tf_{t,d}$.
- Each document is a count vector in $\mathbb{N}^v$: a column below.

<table>
<thead>
<tr>
<th>Term</th>
<th>As You Like It</th>
<th>Twelfth Night</th>
<th>Julius Caesar</th>
<th>Henry V</th>
</tr>
</thead>
<tbody>
<tr>
<td>battle</td>
<td>1</td>
<td>1</td>
<td>8</td>
<td>15</td>
</tr>
<tr>
<td>soldier</td>
<td>2</td>
<td>2</td>
<td>12</td>
<td>36</td>
</tr>
<tr>
<td>fool</td>
<td>37</td>
<td>58</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>clown</td>
<td>6</td>
<td>117</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Comparing the similarity of documents
Comparing the similarity of documents

- Author attribution / plagiarism detection / document de-duplication
- Clustering documents into categories
- Recommendation systems
Term-document matrix

- Two documents are similar if their vectors are similar

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<td>117</td>
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<td>0</td>
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</table>
The words in a term-document matrix

- Each word is a count vector in $\mathbb{N}^D$: a row below

<table>
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<td>117</td>
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<td>0</td>
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</table>
The words in a term-document matrix

- Two **words** are similar if their vectors are similar

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<td>0</td>
</tr>
</tbody>
</table>
If we consider optometrist and eye-doctor we find that, as our corpus of utterances grows, these two occur in almost the same environments. In contrast, there are many sentence environments in which optometrist occurs but lawyer does not...

It is a question of the relative frequency of such environments, and of what we will obtain if we ask an informant to substitute any word he wishes for optometrist (not asking what words have the same meaning).

These and similar tests all measure the probability of particular environments occurring with particular elements... If A and B have almost identical environments we say that they are synonyms.

–Zellig Harris (1954)

“You shall know a word by the company it keeps!”
–John Firth (1957)
Word-word matrix

• Turney and Pantel (2010) *From Frequency to Meaning*: If units of text have similar vectors in a text-frequency matrix then they tend to have similar meanings.
Defining a co-occurrence matrix
DIRT

Lin and Panel (2001) operationalize the Distributional Hypothesis using dependency relationships to define similar environments.

**Duty** and **responsibility** share a similar set of dependency contexts in large volumes of text:

<table>
<thead>
<tr>
<th>modified by adjectives</th>
<th>objects of verbs</th>
</tr>
</thead>
<tbody>
<tr>
<td>additional, administrative, assigned, assumed, collective, congressional, constitutional ...</td>
<td>assert, assign, assume, attend to, avoid, become, breach ...</td>
</tr>
</tbody>
</table>
Define a co-occurrence matrix

sugar, a sliced lemon, a tablespoonful of their enjoyment. Cautiously she sampled her first well suited to programming on the digital for the purpose of gathering data and apricot preserve or jam, a pinch each of, pineapple and another fruit whose taste she likened pineapple In finding the optimal R-stage policy from information necessary for the study authorized in the

<table>
<thead>
<tr>
<th></th>
<th>aardvark</th>
<th>computer</th>
<th>data</th>
<th>pinch</th>
<th>result</th>
<th>sugar</th>
</tr>
</thead>
<tbody>
<tr>
<td>apricot</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>pineapple</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>digital</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>information</td>
<td>0</td>
<td>1</td>
<td>6</td>
<td>0</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

Figure 19.2 Co-occurrence vectors for four words, computed from the Brown corpus, showing only six of the dimensions (hand-picked for pedagogical purposes). Note that a real vector would be vastly more sparse.
Term-context matrix for word similarity

- Two **words** are similar in meaning if their context vectors are similar

|            | aardvark | computer | data | pinch | result | sugar | ...
|------------|----------|----------|------|-------|--------|-------|-----
| apricot    | 0        | 0        | 0    | 1     | 0      | 1     |     |
| pineapple  | 0        | 0        | 0    | 1     | 0      | 1     |     |
| digital    | 0        | 2        | 1    | 0     | 1      | 0     |     |
| information| 0        | 1        | 6    | 0     | 4      | 0     |     |
The word-word or word-context matrix

• Instead of entire documents, use smaller contexts
  • Paragraph
  • Window of ± 4 words
• A word is now defined by a vector over counts of context words
• Instead of each vector being of length D
• Each vector is now of length $|V|$
• The word-word matrix is $|V| \times |V|$
Word-word matrix

• We showed only 4x6, but the real matrix is 50,000 x 50,000
  • So it’s very sparse
    • Most values are 0.
  • That’s OK, since there are lots of efficient algorithms for sparse matrices.

• The size of windows depends on your goals
  • The shorter the windows, the more **syntactic** the representation
    ± 1-3 very syntacticy
  • The longer the windows, the more **semantic** the representation
    ± 4-10 more semanticy
Related versus similar

• **Related** words
  • They are typically nearby each other.
  • *wrote* is related to words like *book* or *poem*.

• **Similar** words
  • They have similar neighbors.
  • *wrote* is similar to words like *said* or *remarked*. 
Vector Semantics

Measuring similarity: the cosine
Measuring similarity

- Given 2 target words $v$ and $w$
- We’ll need a way to measure their similarity.
- Most measure of vectors similarity are based on the:
  - **Dot product** or **inner product** from linear algebra

\[
\text{dot-product}(\vec{v}, \vec{w}) = \vec{v} \cdot \vec{w} = \sum_{i=1}^{N} v_i w_i = v_1 w_1 + v_2 w_2 + \ldots + v_N w_N
\]

- High when two vectors have large values in same dimensions.
- Low (in fact 0) for **orthogonal vectors** with zeros in complementary distribution.
Problem with dot product

\[
\text{dot-product}(\vec{v}, \vec{w}) = \vec{v} \cdot \vec{w} = \sum_{i=1}^{N} v_iw_i = v_1w_1 + v_2w_2 + \ldots + v_Nw_N
\]

• Dot product is longer if the vector is longer. Vector length:

\[
|\vec{v}| = \sqrt{\sum_{i=1}^{N} v_i^2}
\]

• Vectors are longer if they have higher values in each dimension
• That means more frequent words will have higher dot products
• That’s bad: we don’t want a similarity metric to be sensitive to word frequency
Solution: cosine

- Just divide the dot product by the length of the two vectors!

\[
\frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}
\]

- This turns out to be the cosine of the angle between them!

\[
\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta
\]

\[
\frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \cos \theta
\]
Cosine for computing similarity

Dot product

Unit vectors

\[ \cos(\vec{v}, \vec{w}) = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}| |\vec{w}|} = \frac{\vec{v}}{|\vec{v}|} \cdot \frac{\vec{w}}{|\vec{w}|} = \frac{\sum_{i=1}^{N} v_i w_i}{\sqrt{\sum_{i=1}^{N} v_i^2} \sqrt{\sum_{i=1}^{N} w_i^2}} \]

\(v_i\) is the PPMI value for word \(v\) in context \(i\)

\(w_i\) is the PPMI value for word \(w\) in context \(i\).

\(\text{Cos}(\vec{v}, \vec{w})\) is the cosine similarity of \(\vec{v}\) and \(\vec{w}\)
Cosine as a similarity metric

- -1: vectors point in opposite directions
- +1: vectors point in same directions
- 0: vectors are orthogonal

- Raw frequency is non-negative, so cosine ranges between 0 and 1
\[
\cos(\vec{v}, \vec{w}) = \frac{\vec{v} \cdot \vec{w}}{||\vec{v}|| \cdot ||\vec{w}||} = \frac{\sum_{i=1}^{N} v_i w_i}{\sqrt{\sum_{i=1}^{N} v_i^2} \sqrt{\sum_{i=1}^{N} w_i^2}}
\]

Which pair of words is more similar?

<table>
<thead>
<tr>
<th></th>
<th>large</th>
<th>data</th>
<th>computer</th>
</tr>
</thead>
<tbody>
<tr>
<td>apricot</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>digital</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>information</td>
<td>1</td>
<td>6</td>
<td>1</td>
</tr>
</tbody>
</table>

\[
\cos(\text{apricot, information}) = \frac{2 + 0 + 0}{\sqrt{2 + 0 + 0} \sqrt{1 + 36 + 1}} = \frac{2}{\sqrt{2} \sqrt{38}} = .23
\]

\[
\cos(\text{digital, information}) = \frac{0 + 6 + 2}{\sqrt{0 + 1 + 4} \sqrt{1 + 36 + 1}} = \frac{8}{\sqrt{38} \sqrt{5}} = .58
\]

\[
\cos(\text{apricot, digital}) = \frac{0 + 0 + 0}{\sqrt{1 + 0 + 0} \sqrt{0 + 1 + 4}} = 0
\]
Visualizing vectors and angles

<table>
<thead>
<tr>
<th></th>
<th>large</th>
<th>data</th>
</tr>
</thead>
<tbody>
<tr>
<td>apricot</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>digital</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>information</td>
<td>1</td>
<td>6</td>
</tr>
</tbody>
</table>

Dimension 1: ‘large’

Dimension 2: ‘data’
Clustering vectors to visualize similarity in co-occurrence matrices

Rohde et al. (2006)
Other possible similarity measures

\[
\begin{align*}
\text{sim}_{\text{cosine}}(\vec{v}, \vec{w}) &= \frac{\vec{v} \cdot \vec{w}}{||\vec{v}|| ||\vec{w}||} = \frac{\sum_{i=1}^{N} v_i \times w_i}{\sqrt{\sum_{i=1}^{N} v_i^2} \sqrt{\sum_{i=1}^{N} w_i^2}} \\
\text{sim}_{\text{Jaccard}}(\vec{v}, \vec{w}) &= \frac{\sum_{i=1}^{N} \min(v_i, w_i)}{\sum_{i=1}^{N} \max(v_i, w_i)} \\
\text{sim}_{\text{Dice}}(\vec{v}, \vec{w}) &= \frac{2 \times \sum_{i=1}^{N} \min(v_i, w_i)}{\sum_{i=1}^{N} (v_i + w_i)} \\
\text{sim}_{\text{JS}}(\vec{v} \mid \vec{w}) &= D(\vec{v} \mid \frac{\vec{v} + \vec{w}}{2}) + D(\vec{w} \mid \frac{\vec{v} + \vec{w}}{2})
\end{align*}
\]
Vector Semantics

Measuring similarity: the cosine
Using syntax to define a word’s context

- Zellig Harris (1968)
  “The meaning of entities, and the meaning of grammatical relations among them, is related to the restriction of combinations of these entities relative to other entities”

- Two words are similar if they have similar syntactic contexts

Duty and responsibility have similar syntactic distribution:

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Co-occurrence vectors based on syntactic dependencies

Dekang Lin, 1998 “Automatic Retrieval and Clustering of Similar Words”

• Each dimension: a context word in one of R grammatical relations
  • Subject-of- “absorb”
• Instead of a vector of $|V|$ features, a vector of $R|V|$
• Example: counts for the word cell :

<table>
<thead>
<tr>
<th></th>
<th>subj-of, absorb</th>
<th>subj-of, adapt</th>
<th>subj-of, behave</th>
<th>subj-of, inside</th>
<th>subj-of, into</th>
<th>nmod-of, abnormality</th>
<th>nmod-of, anemia</th>
<th>nmod-of, architecture</th>
<th>obj-of, attack</th>
<th>obj-of, call</th>
<th>obj-of, come from</th>
<th>obj-of, decorate</th>
<th>nmod, bacteria</th>
<th>nmod, body</th>
<th>nmod, bone marrow</th>
</tr>
</thead>
<tbody>
<tr>
<td>cell</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>16</td>
<td>30</td>
<td>3</td>
<td>8</td>
<td>1</td>
<td>6</td>
<td>11</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
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</table>
Syntactic dependencies for dimensions

- Alternative (Padó and Lapata 2007):
  - Instead of having a $|V| \times R|V|$ matrix
  - Have a $|V| \times |V|$ matrix
  - But the co-occurrence counts aren’t just counts of words in a window
  - But counts of words that occur in one of R dependencies (subject, object, etc).
  - So $M(\text{“cell”,”absorb”}) = \text{count(subj(cell,absorb))} + \text{count(obj(cell,absorb))} + \text{count(pobj(cell,absorb))}$, etc.
TF-IDF

- **tf-idf** (that’s a hyphen not a minus sign)
- The combination of two factors
  - **Term frequency** (Luhn 1957): frequency of the word (can be logged)
  - **Inverse document frequency** (IDF) (Sparck Jones 1972)
    - N is the total number of documents
    - \( df_i \) = “document frequency of word \( i \)”
      - = # of documents with word \( i \)
    - \( \frac{N}{df_i} \)
    - \( idf_i = \log \left( \frac{N}{df_i} \right) \)
  - \( w_{ij} = \text{word } i \text{ in document } j \)
    - \( w_{ij} = tf_{ij} \cdot idf_i \)
TF-IDF

• TF-IDF not generally used for word-word similarity
• But is by far the most common weighting when we are considering the relationship of words to documents
Vector Semantics

Positive Pointwise Mutual Information (PPMI)
Problem with raw counts

• Raw word frequency is not a great measure of association between words
  • It’s very skewed
    • “the” and “of” are very frequent, but maybe not the most discriminative
• We’d rather have a measure that asks whether a context word is particularly informative about the target word.
  • Positive Pointwise Mutual Information (PPMI)
Pointwise Mutual Information

Pointwise mutual information:
Do events $x$ and $y$ co-occur more than if they were independent?

$$\text{PMI}(X,Y) = \log_2 \frac{P(x,y)}{P(x)P(y)}$$

PMI between two words: (Church & Hanks 1989)
Do words $x$ and $y$ co-occur more than if they were independent?

$$\text{PMI}(\text{word}_1, \text{word}_2) = \log_2 \frac{P(\text{word}_1, \text{word}_2)}{P(\text{word}_1)P(\text{word}_2)}$$
PMI applied to dependency relations

Hindle, Don. 1990. Noun Classification from Predicate-Argument Structure. ACL

<table>
<thead>
<tr>
<th>Object of “drink”</th>
<th>Count</th>
<th>PMI</th>
</tr>
</thead>
<tbody>
<tr>
<td>tea</td>
<td>2</td>
<td>11.8</td>
</tr>
<tr>
<td>liquid</td>
<td>2</td>
<td>10.5</td>
</tr>
<tr>
<td>wine</td>
<td>2</td>
<td>9.3</td>
</tr>
<tr>
<td>anything</td>
<td>3</td>
<td>5.2</td>
</tr>
<tr>
<td>it</td>
<td>3</td>
<td>1.3</td>
</tr>
</tbody>
</table>

- “Drink it” more common than “drink wine”
- But “wine” is a better “drinkable” thing than “it”
Positive Pointwise Mutual Information

- PMI ranges from $-\infty$ to $+\infty$
- But the negative values are problematic
  - Things are co-occurring less than we expect by chance
  - Unreliable without enormous corpora
    - Imagine $w_1$ and $w_2$ whose probability is each $10^{-6}$
    - Hard to be sure $p(w_1,w_2)$ is significantly different than $10^{-12}$
- Plus it’s not clear people are good at “unrelatedness”
- So we just replace negative PMI values by 0
- Positive PMI (PPMI) between word1 and word2:
  \[
  \text{PPMI}(\text{word}_1, \text{word}_2) = \max \left( \log_2 \frac{P(\text{word}_1, \text{word}_2)}{P(\text{word}_1)P(\text{word}_2)}, 0 \right)
  \]
Computing PPMI on a term-context matrix

- Matrix $F$ with $W$ rows (words) and $C$ columns (contexts)
- $f_{ij}$ is the number of times $w_i$ occurs in context $c_j$

\[
\begin{align*}
    p_{ij} &= \frac{f_{ij}}{\sum_{i=1}^{W} \sum_{j=1}^{C} f_{ij}} \\
    p_{i*} &= \frac{\sum_{j=1}^{C} f_{ij}}{\sum_{i=1}^{W} \sum_{j=1}^{C} f_{ij}} \\
    p_{*j} &= \frac{\sum_{i=1}^{W} f_{ij}}{\sum_{i=1}^{W} \sum_{j=1}^{C} f_{ij}} \\
    pmi_{ij} &= \log_2 \frac{p_{ij}}{p_{i*} p_{*j}} \\
    ppmi_{ij} &= \begin{cases} 
    pmi_{ij} & \text{if } pmi_{ij} > 0 \\
    0 & \text{otherwise}
    \end{cases}
\end{align*}
\]
\[ p_{ij} = \frac{f_{ij}}{\sum_{i=1}^{W} \sum_{j=1}^{C} f_{ij}} \]

**Count(w,context)**

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<tr>
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<th>result</th>
<th>sugar</th>
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<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>pineapple</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
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<td>1</td>
<td>0</td>
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<td>0</td>
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<tr>
<td>information</td>
<td>1</td>
<td>6</td>
<td>0</td>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ p(w=\text{information},c=\text{data}) = \frac{6}{19} = .32 \]

\[ p(w=\text{information}) = \frac{11}{19} = .58 \]

\[ p(c=\text{data}) = \frac{7}{19} = .37 \]

\[ p(w_i) = \frac{\sum_{j=1}^{C} f_{ij}}{N} \]

\[ p(c_j) = \frac{\sum_{i=1}^{W} f_{ij}}{N} \]

**p(w,context)**

<table>
<thead>
<tr>
<th></th>
<th>computer</th>
<th>data</th>
<th>pinch</th>
<th>result</th>
<th>sugar</th>
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<tbody>
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**p(w)**

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</tbody>
</table>

\[ p(\text{context}) = \frac{16}{19} = .37 \]
\[ pmi_{ij} = \log_2 \frac{p_{ij}}{p_i p_j} \]

<table>
<thead>
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<th>result</th>
<th>sugar</th>
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<td>0.11</td>
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</table>

- \( pmi(\text{information, data}) = \log_2 \left( \frac{0.32}{(0.37 \times 0.58)} \right) = 0.58 \)

\[ \text{PPMI}(w, \text{context}) \]

<table>
<thead>
<tr>
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(0.57 using full precision)
Weighting PMI

• PMI is biased toward infrequent events
  • Very rare words have very high PMI values

• Two solutions:
  • Give rare words slightly higher probabilities
  • Use add-one smoothing (which has a similar effect)
Weighting PMI: Giving rare context words slightly higher probability

• Raise the context probabilities to $\alpha = 0.75$:

$$\text{PPMI}_\alpha(w, c) = \max(\log_2 \frac{P(w, c)}{P(w)P_\alpha(c)}, 0)$$

$$P_\alpha(c) = \frac{\text{count}(c)^\alpha}{\sum_c \text{count}(c)^\alpha}$$

• This helps because $P_\alpha(c) > P(c)$ for rare $c$

• Consider two events, $P(a) = .99$ and $P(b)=.01$

$$P_\alpha(a) = \frac{.99^{0.75}}{.99^{0.75} + .01^{0.75}} = .97$$

$$P_\alpha(b) = \frac{.01^{0.75}}{.01^{0.75} + .01^{0.75}} = .03$$
Use Laplace (add-1) smoothing
### Add-2 Smoothed Count($w$,context)

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### $p(w,\text{context})$ [add-2]

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### $p(w)$

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### $p(\text{context})$

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# PPMI versus add-2 smoothed PPMI

## PPMI(w,context)

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## PPMI(w,context) [add-2]

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Vector Semantics
Evaluating similarity
Evaluating similarity

- **Extrinsic** (task-based, end-to-end) Evaluation:
  - Question Answering
  - Spell Checking
  - Essay grading

- **Intrinsic Evaluation**:
  - Correlation between algorithm and human word similarity ratings
    - Wordsim353: 353 noun pairs rated 0-10. \( sim(\text{plane}, \text{car}) = 5.77 \)
  - Taking TOEFL multiple-choice vocabulary tests
    - *Levied* is closest in meaning to:
      imposed, believed, requested, correlated
Summary

• Distributional (vector) models of meaning
  • Sparse (PPMI-weighted word-word co-occurrence matrices)
  • Dense: Next lecture!